Investigating the Explore/Exploit Trade-off in Adult Causal Inferences

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Abstract

We explore how adults learn counterintuitive causal relationships, and whether they discover hypotheses by revising their beliefs incrementally. We examined how adults learned a novel and unusual causal rule when presented with data that initially appeared to conform to a simpler, more salient rule. Adults watched a video of several blocks placed sequentially on a blicket detector, and were then asked to determine the underlying causal structure. In the near condition the true rule was complex, but could be found by making incremental improvements to the simple and salient initial hypothesis. The distant condition was governed by a simpler rule, but to adopt that rule participants had to set aside their initial beliefs, rather than revising them incrementally. Adults performed better in the near condition, despite this rule being more complex, providing some of the first evidence for an explore-exploit trade-off in inference, analogous to the trade-off in active learning.

Keywords: causality, Bayesian inference, hypothesis search, process model

Background

Any time we make plans, predict the future, or attempt to understand why events occurred in the past, we are relying on causal knowledge. In acquiring this knowledge, we must draw conclusions from sparse, noisy, and ambiguous evidence. To make sense of these kinds of data, we must have abstract beliefs, sometimes described as overhypotheses, about what kinds of causal relationships are more plausible than others. We begin to form these hypotheses at an early age, with causal thinking showing signs of emergence even in infancy (Sobel & Kirkham, 2006; 2007; Walker & Gopnik, 2014). By adulthood, our frameworks for interpreting causal phenomena become much more complex and able to accommodate diverse areas of knowledge (Kemp, Goodman, & Tenenbaum, 2007).

Despite their usefulness, sometimes these causal expectations can lead us astray, as in the case where we encounter a new causal relationship that is rare or strange by the standards of our past experience. For instance, we might expect that either of two switches will turn on a lamp, when in fact the lamp turns on when the switches are in matched positions. While our causal learning process is generally accurate and adaptive (e.g., Griffiths & Tenenbaum, 2005), in the current paper we claim – in the spirit of previous "rational process" models (e.g. Sanborn, Griffiths, & Navarro, 2010) – that human causal beliefs are updated in a limited or local fashion that is efficient but subject to systematic failures under certain conditions. This is especially true when the true causal structure is distant from our initial hypothesis in some hypothesis space. Suppose you break out in a rash every time you buy your favourite candy bar from a vending machine. After searching for the proper cause, you would probably conclude that you are allergic to the candy as soon as it comes to mind. You may be unlikely to consider that you are actually reacting to the coins used to purchase the candy bar, even if this is indeed the case. In this instance, inferring the proper cause requires looking beyond the most obvious solution, which may be difficult to accomplish.

Bayesian Models of Causal Inference

Several researchers have attempted to explain learning of novel causal relationships using hierarchical Bayesian models of inference (e.g. Griffiths, Sobel, Tenenbaum, & Gopnik, 2011; Griffiths, Kemp, & Tenenbaum, 2008). Recent evidence demonstrates that both adults and children can successfully modify their causal beliefs in light of new and surprising evidence in a manner that suggests they are using a Bayesian inference strategy (e.g., Griffiths, Sobel, Tenenbaum, & Gopnik, 2011; Lucas, Bridgers, Griffiths, & Gopnik, 2014). Through this process, learners can also create and update higher-level models of how causal relationships operate in general. Regardless of whether human cognition functions exactly in this manner, hierarchical Bayesian models have been shown to accurately predict causal learning (Kemp, Goodman, & Tenenbaum, 2007; Lu, Yuille, Lijeholm, Cheng, & Holyoak, 2006; Lucas & Griffiths, 2010).

However, Bayesian inference is often intractable in practice for complex problems, which may preclude people from solving them as an ideal Bayesian agent would. Multi-variable causal inference entails an enormous space of possible hypotheses; inferring the nature of a relationship between k binary-valued causes and one effect entails a hypothesis space with roughly 2^{2^k} entries. As for the process by which people might make approximately Bayesian inferences given limited resources, empirical phenomena such as order effects offer hints. If learners are making inferences from a complete set of data, as traditional Bayesian models assume, then they should not be influenced by the order in which stimuli are presented. Nevertheless, studies with inference problems show that phenomena such as order effects and anchoring are prevalent (Danks & Schwartz, 2006; Sanborn, Griffiths,

& Navarro, 2010). This suggests that people arrive at solutions by considering a small number of hypotheses at any single moment in time, and updating or replacing them sequentially as more data become available – sometimes losing information and leading to small but systematic errors. More recently, Bayesian process models have been proposed to explain these patterns of errors by drawing analogies to inference algorithms that permit tractable and efficient inference in applied statistics and machine learning. (Abbott, Hamrick, & Griffiths, 2013; Shi, Griffiths, Feldman, & Sanborn, 2010).

Although existing Bayesian models can accommodate certain biases, they may not fully account for adults' relative difficulties in learning more unusual types of causal relationships. Specifically, Lucas and colleagues (2014) found that young children were more likely than adults to discover an unusual conjunctive causal relationship. Children and adults were tasked with inferring a causal principle after viewing a machine that activated when certain blocks or block combinations were placed on top of it. Even after viewing evidence that blocks only activated the machine in specific pairs (and not individually), adults had more difficulty than children with generalizing this principle to new blocks. One possibility for this finding is that adults are more biased by prior experiences—as they have observed that conjunctive relationships are relatively rare—which leads them to demand strong evidence before they infer a conjunctive relationship is present. Indeed, the mere fact that adults *have* more prior experience means that adults have developed a wider range of overhypotheses related to the kinds of causal relationships that are likely to exist. If cognition operates via Bayesian principles, there are conceivably instances in which rigid commitment to a prior may preclude learners from uncovering the true nature of a causal relationship. However, this may not apply in novel causal situations with which adults have limited experience. Moreover, adults are cognitively different than children beyond simply having more experience, so differences in causal reasoning may in fact be the by-product of some developmental change.

The Explore-Exploit Trade-off in Inference

As an alternative to simply having different priors, adults' relative difficulty with conjunctive causal relationships may be explained in terms of the *process* by which they explore and weigh new hypotheses in light of their current beliefs. It is typically not possible to evaluate all potential hypotheses (of which there may be an infinite number). As a result, we might expect that people approximate the full posterior by examining a subset of the full range of hypotheses, and, in the extreme, considering just one at a time. For example, in certain causal learning situations, children and adults might employ "win-stay, lose-shift" (or "lose-sample") strategies, whereby consistent evidence may reinforce the hypothesis over time and inconsistent evidence may trigger belief revision (Bonawitz, Denison, Gopnik, & Griffiths, 2014). These strategies, as well as other causal learning estimation techniques, are often modelled using Monte Carlo methods that update sequentially and incrementally. These methods allow hypotheses to be revised by sampling from the posterior, without computing the posterior distribution in its entirety.

Markov chain Monte Carlo sampling algorithms in particular exhibit a degree of stickiness or inertia, in which they hew more closely to their initial hypotheses than a truly optimal Bayesian learner would. This family of models predicts that individuals will tend toward inferences that are similar to their prior beliefs. For example, one study showed that when people made inferences about a causal system, they tended toward solutions that required the fewest single edits to their initial hypothesis, where a single edit is an addition, subtraction, or reversal of a causal link (Bramley, Dayan, & Lagnado, 2015). This idea has recently been shown to explain classical anchoring phenomena (Lieder, Griffiths, & Goodman, 2012). Therefore, causal process models can account for multiple limitations on causal learning; learners can be constrained not only by one's priors, but also the similarity of candidate hypotheses to their current beliefs, perhaps precluding them from even finding hypotheses that are too distant.

Gopnik and colleagues (Gopnik, Griffiths, & Lucas, 2015) recently conjectured that these inference by sampling models might explain developmental differences in causal learning, suggesting that young children's relative cognitive flexibility may be advantageous when searching for solutions to causal problems. Greater flexibility may shield children from cognitive biases present in adults, which would explain children's relatively high performance in Lucas and colleagues' (2014) study. Therefore, these findings could reflect a cognitive tradeoff in development that affects how children and adults search through hypotheses. When presented with a wide range of possibilities, individuals must often decide whether to employ a general, shallow search or a narrow, deep one. This is related to the explore-exploit tradeoff, whereby decision-makers must allocate cognitive resources to either exploit previous knowledge or explore alternatives (Sutton & Barto, 1998). From a developmental standpoint, adults may be more inclined to exploit than children are—and less likely to explore hypotheses with a greater edit distance from the current hypothesis—thereby increasing efficiency but potentially limiting access to unusual alternatives.

Thus, the inferential explore-exploit trade-off may have interesting implications for the process of selecting between competing hypotheses. This selection process has been modelled using Bayesian algorithms for both children and adults (Bonawitz, Denison, Gopnik, & Griffiths, 2014; Denison, Bonawitz, Gopnik, & Griffiths, 2013; Lieder, Griffiths, & Goodman, 2012; Sanborn, Griffiths, & Navarro, 2010), but relatively little previous work has examined changes in exploration and exploitation. As one possible example of how hypothesis search may reflect an exploitation bias, researchers have likened problem-solving development to simulated annealing; just as the heating and gradual cooling of a metal can increase its malleability, so can a gradual "cooling" of an inference method corresponding to an increasingly conservative search policy lead to better inferences (Gopnik, Griffiths, & Lucas, 2015; Lucas, Bridgers, Griffiths, & Gopnik, 2014). For instance, while young children may use high-temperature searches, considering a wide range of hypotheses with relatively equal probability, adults' searches are "cooler" and more narrow in scope. Although commitment to priors may still play a crucial role, simulated annealing allows us to examine which types of hypotheses are considered. High-temperature searches are more likely to discard adequate hypotheses, but may allow individuals to escape local optima and discover unlikely solutions that are potentially better. In contrast, low-temperature searches can quickly converge to good solutions if fewer low-probability edits are required to get there, but may otherwise get trapped in local optima. With this in mind, adults may have more difficulty with certain unusual causal relationships because their search is too focused and too close to their initial guesses to accommodate distant ideas. While Lucas and colleagues (2014) suggested the possibility that children and adults explore the hypothesis space differently, they did not distinguish it from the idea that adults simply have stronger priors than children. While both possibilities would result in a tendency for adults to disfavour unlikely solutions, the ideas function fundamentally differently.

The purpose of our current studies is to test the hypothesis that belief updating in adults is *exploitation-biased*. To accomplish this, we designed a task that encouraged participants to generate a particular initial hypothesis about a novel causal relationship. Evidence that contradicted this hypothesis was then presented, causing participants to modify their beliefs. The true causal structure took one of two forms corresponding to two experimental conditions. In the *near condition*, the correct causal structure was closer to the initial hypothesis but relatively complex. In the *distant condition*, the correct causal structure was simpler but unrelated to the initial hypothesis. Thus, we hoped to determine the breadth of hypotheses that the participants were willing to entertain. If adults' search process is more exploitation-biased, we should expect the near-hypothesis solution would be more easily found than the distant one, even if both rules are a priori equally unlikely. However, if adults' failure to infer unlikely causal relationships is simply due to the low prior probability that they place on these relationships, then they should be equally unlikely to consider either solution.

Experiment 1: Investigating the Explore-Exploit Tradeoff in Inference

Participants Participants were 90 adult US residents, recruited through Amazon Mechanical Turk and paid a base rate of $1 for their time. An additional $1 bonus was given to the top 10% performers as an additional incentive. Participants were divided randomly among near (n = 45) and distant (n = 45) conditions. Six participants from the near condition and seven from the distant condition were excluded due to failure to correctly answer attention manipulation tasks.

Materials and Procedure The methods used in this study are similar to those used in previous blicket tasks (e.g. Gopnik & Sobel, 2000), except that animated video stimuli were presented online using Qualtrics survey software (similar to Buchsbaum et al., 2012). Participants were asked to examine several blocks and determine which blocks are blickets. They were informed that blickets are blocks that activate the blicket detector, and were shown a video of an animated blicket detector activating and not activating. Participants then watched a five-minute animation depicting 20 blocks being consecutively placed onto the blicket detector. If the block was a blicket, the detector lit up and a sound played.

Whether a block was a blicket depended on specific aspects of the block pattern. Each block had a coloured background (red or blue) and several small red or blue triangles in a fixed pattern (see Figure 1). The block pattern was such that the background colour was the most obvious and visually striking feature. For the first 15 blocks (the *rule-consistent* blocks), the background colour appeared to determine whether the blocks activated the machine—i.e. blocks with one background colour consistently activated the machine, while the others did not. Inspired by an experimental manipulation in Williams and Lombrozo (2010; 2013), this was designed to lead participants to an initial causal hypothesis based on the objects' most salient feature. The final five blocks (the *rule-violating* blocks), however, violated this initial hypothesis; the blocks that did and did not activate the machine had the opposite background colour as before. Thus participants needed to modify their initial hypothesis to capture the optimal solution. In the near condition, participants saw 11 blickets (3 rule-violating) and 9 non-blickets (2 rule-violating). In the distant condition participants saw 10 blickets (2 rule-violating) and 10 non-blickets (3 rule-violating).

The true rule separating blickets from non-blickets varied based on condition. This true rule determined whether a block was a blicket 100% of the time. In the near condition, the background colour was related to whether a block was a blicket, whereas in the distant condition the background colour was unrelated. Each block had five binary features (Figure 1), which could vary by colour on each block (background, corners, centre-left triangle, centre-right triangle, and border), giving a total of 32 different colour combinations. In the near condition, blocks were blickets based on a combination of the background colour and the colour of two secondary features. In the distant condition, only the colour of these two secondary features determined whether a block was a blicket, while the background colour was irrelevant.

Thus, the five features could be labeled as follows: one primary feature (A), two relevant secondary features (B and C), and two irrelevant secondary features (D and E). In the distant condition, the optimal rule for determining whether a block is a blicket—that is, the simplest rule that perfectly explains the data—can be written as $R = (B \cap \neg C) \cup (\neg B \cap C)$, whereas the optimal rule in the near condition can be written as $R = (A \cap \neg B) \cup (\neg A \cap \neg C)$. In the near condition, there

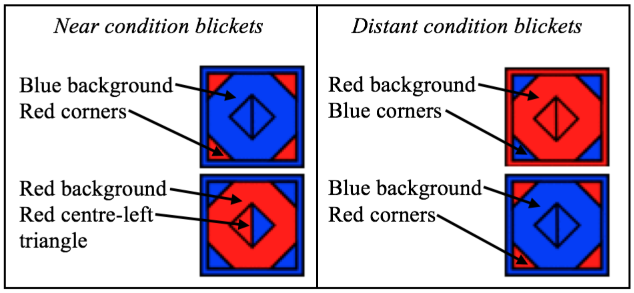

Figure 1: Examples of blickets in the near condition (left) and the distant condition (right).

is a consistently-improving path of single edits to transition from the initial hypothesis, $R = A$, to the correct rule, where a single edit consists of adding or subtracting a variable or changing an operator (e.g. changing $R = A$ to $R = A \cap \neg B$; Goodman & Tenenbaum, 2008 use a similar approach for searching a hypothesis space). In the distant condition, the single-edit path to the correct rule requires edits that initially worsen the hypothesis (e.g. removing A as a relevant variable). If adults use a Bayesian single-edit search process with an exploit bias, participants should be less likely to abandon $R = A$, and thus should perform more poorly in the distant condition, where $R = A$ is the local optimum.

Following the blicket presentation, participants saw a blicket rating task, in which they were asked to judge whether a randomized series of eight blocks were blickets. For each block, participants rated how certain they were that it was, or was not, a blicket, on a seven-point Likert scale ranging from "definitely a blicket" to "definitely not a blicket". Blocks were balanced by background colour, blicket/non-blicket status, and whether they had already been presented in the observation stage. Participants received a score between -3 and 3 for each block based on accuracy and certainty, and the sum of these scores determined their final score for this task.

Next, participants completed a forced-choice task, where they were asked to choose which of two blocks was more likely to activate the blicket detector, for a series of four pairs. Blocks were selected randomly such that there were an equal number of rule-consistent and rule-violating blocks, and blocks in each pair differed from each other in background colour and whether they were a blicket. Participants received a point for each correct block judgment.

Afterwards, the participants were asked to describe the causal rule they had inferred. They were then told to imagine that a new rule was suggested by a friend, and asked if they preferred this rule over their own. This rule always represented the correct causal structure. The purpose of this question was to ensure that any differences between the two conditions were not due to participants finding the near rule inherently more plausible or likely than the distant one. The participants' rule preference was measured using a seven-point scale. Finally, each participant received questions to test their task comprehension and an instructional manipulation task

Table 1: Mean scores and standard error for forced-choice task. Total scores range from 0 to 4, and scores for rule-consistent and rule-violating blocks range from 0 to 2.

Condition	Near	Distant
Total score	2.53($\pm$0.10)	2.24($\pm$0.12)
Rule-consistent	1.90($\pm$0.08)	1.82($\pm$0.07)
Rule-violating	0.77($\pm$0.13)	0.42($\pm$0.07)

Table 2: Mean scores and standard error for blicket rating task. Total scores range from -24 to 24, and scores in each sub-category range from -12 to 12.

Condition	Near	Distant
Total score	8.00($\pm$1.04)	4.87($\pm$1.26)
Rule-consistent	9.59($\pm$0.51)	6.39($\pm$0.72)
Rule-violating	-1.59($\pm$1.01)	-1.53($\pm$1.06)

to control for inattention, similar to the one used by Oppenheimer, Meyvis, and Davidenko (2009).

Results and Discussion If adults' strategy for hypothesis search is exploitation-biased, participants in the near condition will perform better on both tasks than those in the distant condition. The results supported our predictions. For the forced-choice task, a 2x2 ANOVA was run with condition (distant/near) and rule consistency (rule-consistent/violating) as factors (see Table 1 for a score summary). Participants in the near condition scored higher than those in the distant condition, $F(1, 84) = 6.46$, $p = .01$, $MSE = 0.26$. Participants also scored higher for rule-consistent blocks, than for rule-violating blocks, $F(1, 84) = 226$, $p < .001$, $MSE = 0.34$ (see Figure 2 for a visual comparison).

For the blicket rating task, a 2x2 mixed ANOVA (condition x rule consistency) was run (see Table 2 for a score summary). The analysis found that participants were much more likely to confidently identify rule-consistent blocks than rule-violating blocks $F(1, 84) = 131$, $p < .001$, $MSE = 15.32$, suggesting that the salience manipulation was effective. Supporting our forced-choice results, there was a marginally significant effect of condition, $F(1, 84) = 3.77$, $p = .06$, $MSE = 11.87$, with a mean score of 7.51 for the near condition and 4.63 for the distant condition (scores ranged from -24 to 24).

Intriguingly, there was also a significant interaction effect, $F(1, 84) = 3.34$, $p = .04$, $MSE = 15.32$. This is a result of participants in the near condition performing better than those in the distant condition on rule-consistent blocks, but equally poorly on rule-violating blocks. To assess whether this interaction was due to differences in confidence for some blocks, an additional 2x2 mixed ANOVA (condition x rule consistency) was run to investigate participants' certainty ratings when evaluating blocks. Participants were more certain of their answers when rating rule-consistent blocks than when

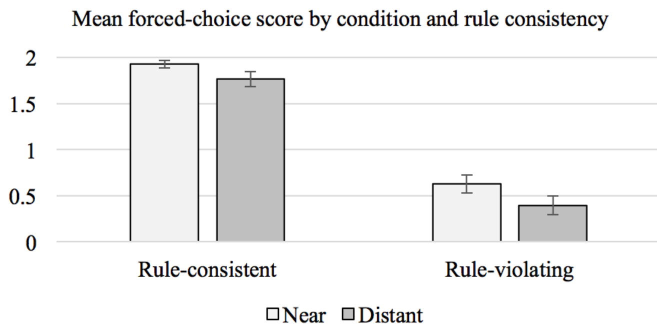

Figure 2: Scores on the forced-choice task as a function of rule consistency. Scores in each category range from 0 to 2.

rating rule-violating blocks, $F(1, 84) = 22.0$, $p < .001$, $MSE = 0.32$. There was also a highly significant interaction effect between condition and rule-consistency, $F(1, 84) = 13.1$, $p < .001$, $MSE = 0.32$, which was driven by participants in the near condition having more certainty for rule-consistent blocks than for rule-inconsistent blocks, suggesting that while participants in the near-condition were better able to correctly categorize both rule-violating and rule-consistent blocks, they were most confident about the latter.

Additional one-sample t-tests examined whether participants scored better than would be expected by chance. For the forced-choice task, participants correctly classified blocks as blickets and non-blickets significantly better than chance in the near condition, $t(42) = 5.82$, $p < .001$, but *not* in the distant condition, $t(42) = 1.31$, $p = 0.20$. In the blicket rating task, however, participants classified blocks better than chance in both the near condition, $t(42) = 7.69$, $p < .001$, and the distant condition $t(42) = 4.13$, $p < .001$.

Finally, we looked at participants' preference for the correct rule over their own. Participants in the distant condition significantly preferred the correct friend's rule over their own rule, t(42) = 4.78, p < .001, while participants in the near condition did not, t(42) = 1.55, p = .13. Participants in the distant condition also preferred the friend's rule significantly more than those in the near condition, t(75) = 2.09, p = .04. This supports our hypothesis that participants in the distant condition had not previously considered the distant rule, rather than that they considered it, but dismissed it as unlikely.

Experiment 2: A priori rule preference

Although the main study compared the extent to which participants preferred the correct rule over their own, it did not examine the rules in both conditions side-by-side. This study investigated adults' a priori preference for either the near or the distant rule without differentiating data. This was to confirm that differences in causal learning and rule preference between conditions in Experiment 1 were not due to an intuitive preference for the near rule before seeing any data.

Participants Participants were 51 adult US residents, recruited through Amazon Mechanical Turk (MTurk) and paid a base rate of $0.50 for their time.

Materials and Procedure As in the previous study, participants were told that blickets were blocks that activated the blicket detector, and saw an animated blicket detector activating and not activating. Unlike the previous study, however, participants only saw one block placed on the machine, causing it to activate. They were then told the two possible rules, and that both rules accurately described this block, but that only one rule was the correct rule for identifying blocks that activate the machine. Participants were asked to choose which rule they thought was more likely to be correct. These rules were identical to the near rule and the distant rule from the previous study, and the blicket that participants saw was chosen from a set of blocks that conformed to both rules. Finally, after selecting a rule, participants explained why they chose that rule and rated their confidence in their decision, ranging from 1 (just guessing) to 7 (completely certain). This confidence rating was turned into a score ranging from -7 (completely certain the near rule is correct) to 7 (completely certain the distant rule is correct) for statistical analysis.

Results and Discussion Of the 51 participants, 22 preferred the near rule and 29 preferred the distant rule, p = .41, exact binomial test. A one-sample t-test demonstrated that the rule preference scores, $M = 0.25$, $SE = 0.50$, did not significantly differ from chance, $t(49) = 0.71$, $p = 0.48$. Thus, participants did not prefer one rule over the other, suggesting that it was not an a priori preference for the near rule driving the results of Experiment 1.

General Discussion

The findings obtained by these studies lend support to the exploitation-biased search hypothesis. We expect that exploitation-biased searches of the hypothesis space will be more likely to discover rules close to the initial hypothesis, and less likely to discover more distant rules that are equally complex. As predicted, participants were more accurate at classifying blocks in the near condition than the distant condition. This is especially notable given that participants in Experiment 2 found both rules equally a priori plausible, which supports that the near rule is at least as complex as the distant rule. This in turn makes it less likely that the differences between conditions can be explained by differently-weighted prior probabilities. Participants performed better in the near condition, where the true rule was arguably more complex, but was comparatively easier to discover from the salient starting point, than in the distant condition, where the true rule was simpler, but where the salient rule was a local optimum. This suggests that adults are searching through their hypothesis space in an exploitation-biased manner.

Nevertheless, participants were better able to identify rule-consistent blocks than rule-violating ones in both tasks. This suggests that the strength of one's priors may still play a role in conjunction with the exploitation bias. However, this difference in performance suggests intriguing future research avenues—in particular, the finding in the blicket rating task that participants in the near condition scored higher than those in the distant condition on rule-consistent but not rule-violating blocks. This seems to be driven largely by participants' relative certainty toward rule-consistent blocks in the near condition, rather than their accuracy at categorizing the blocks (as measured by the forced choice task). Future studies might assess how nearness to an initial hypothesis affects the certainty of judgments of causal relationships.

It is still unclear, however, if these difficulties in discovering certain causal relationships are the result of a developmental process. Consequently, we plan to expand this study to directly compare adults with children, to examine whether children possess these same search-related difficulties. If these findings are the result of a developmental shift toward exploitation-based search strategies, then exploration-oriented children could perform just as well—if not better—than adults in tasks such as those in this study. For this particular study, children should perform equally well in both experimental conditions, or perhaps even better in the distant condition than in the near one. Particularly, this may be the case if children see the near rule as a priori less likely.

In the future, it may be useful to develop a more explicit process model to measure hypothesis distance. Although the near-hypothesis rule is closer to the salient hypothesis, in that adding and subtracting particular predicates always improves the hypothesis toward the correct rule, this may not accurately represent how individuals process locality. In other words, we lack a precise model for how people move between rules, and thus exactly how far $R = (B \cap \neg C) \cup (\neg B \cap C)$ is from $R = A$, and how much harder it is to find $R = (A \cap \neg B) \cup (\neg A \cap \neg C)$. In future experiments, this process model will need to be made more concrete.

Overall, our results demonstrating that adults are able to discover a true causal structure nearer to an initial hypothesis more readily than a distant causal structure of equal or *greater* complexity provides compelling initial evidence for an explore-exploit trade-off in causal inferences. This may help inform future research on how individuals generate new hypotheses about everyday causal interactions.

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